Incorporating sediment-transport capabilities to DSM2

by

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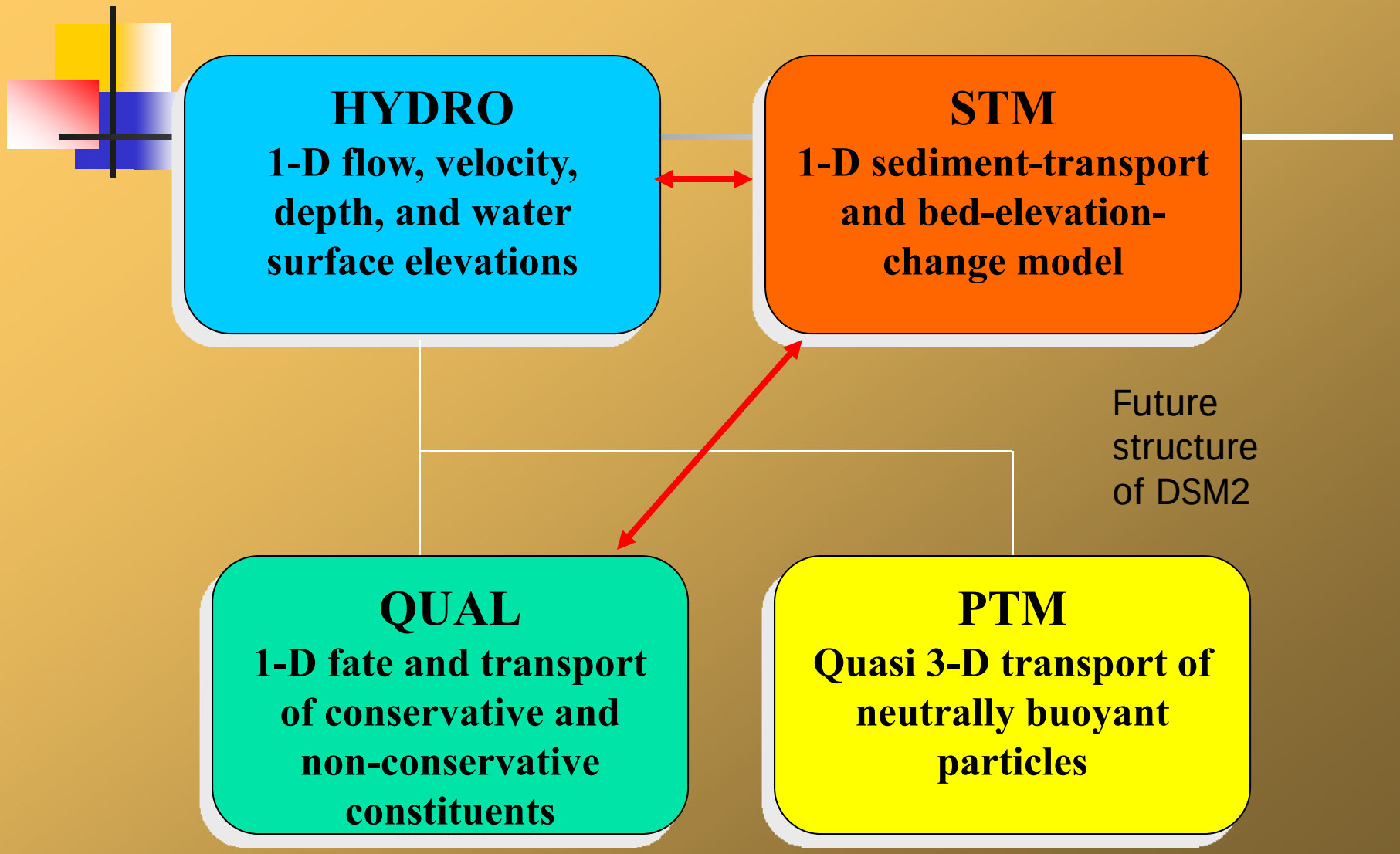
Outline

- Modes of sediment transport
- Mathematical treatment of the problem
- Numerical treatment of the problem
- Proposed work and timeline
- Conclusions

DSM2 and STM


HYDRO
1-D flow, velocity, depth, and water surface elevations
STM
1-D sediment-transport and bed-elevation-change model
Future structure of DSM2
QUAL
1-D fate and transport of conservative and non-conservative constituents
PTM
Quasi 3-D transport of neutrally buoyant particles

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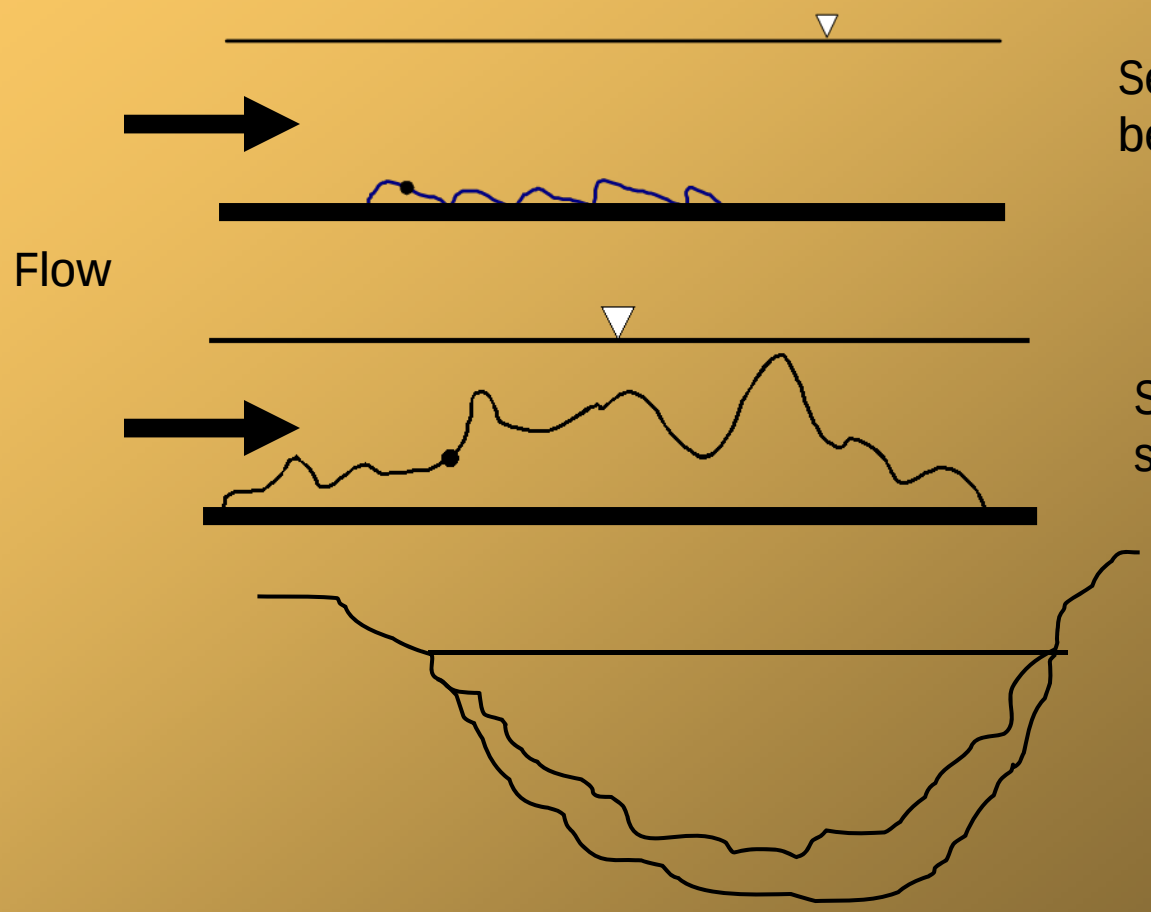
Modes of sediment transport



Sediment transport as bed-load

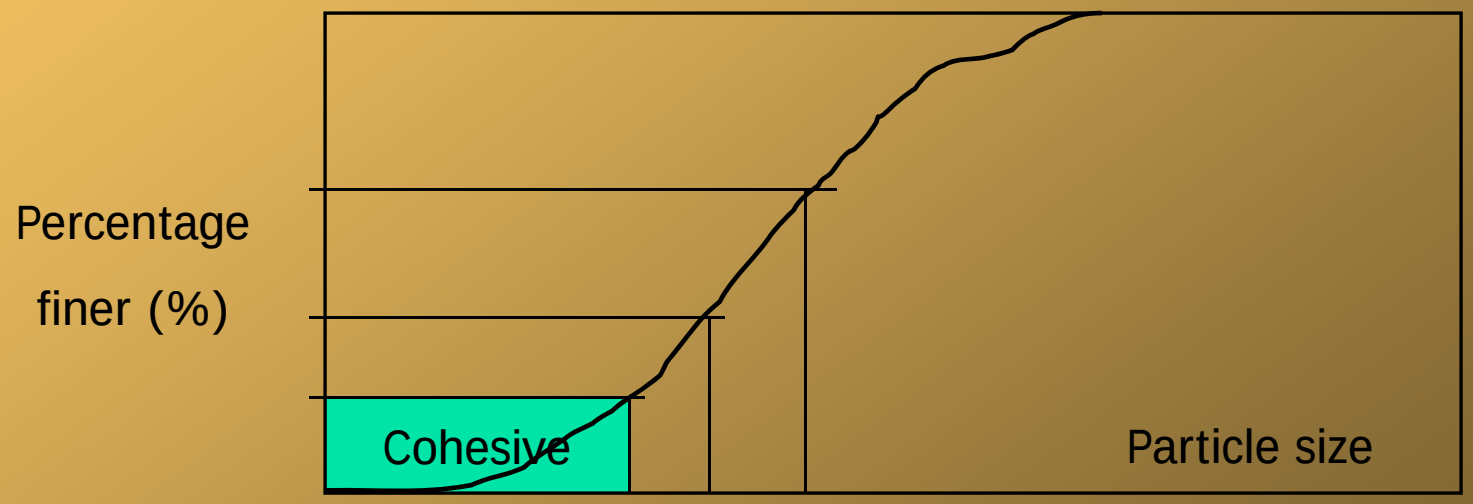
Sediment transport in suspension

Scour of bed and bed changes

Modes of sediment transport

There is a third mode of sediment transport called wash load, whereby very fine particles are transported downstream with little interaction with the sediments in the bed.

During floods, the wash load is deposited in the floodplains. Important in places like the Rio de la Plata, Argentina, where fine sediments come from the Andes.

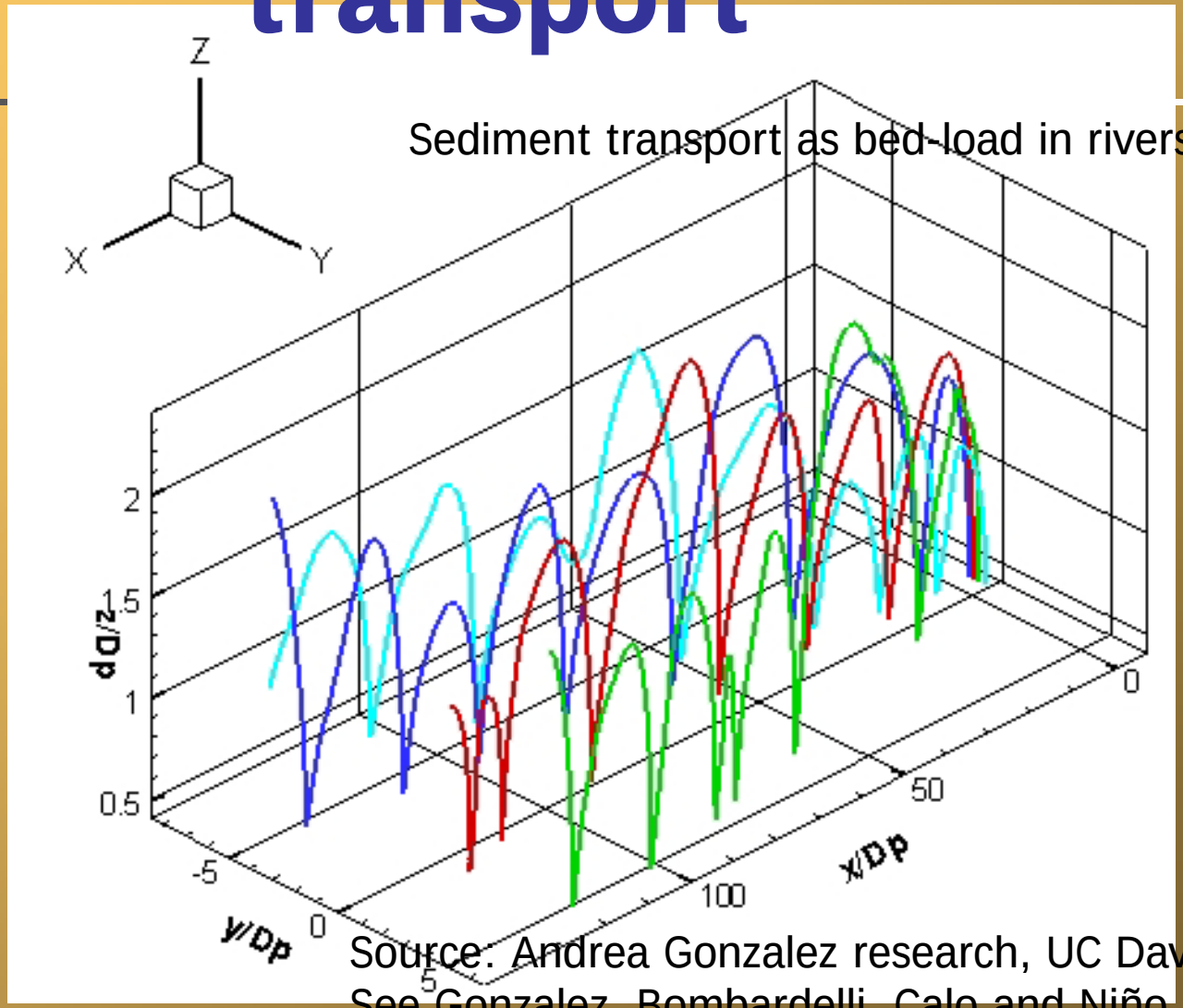


Modes of sediment transport

Sediment transport as bed-load in rivers

Source: Prof. Dietrich's website

Modes of sediment transport



Source: Andrea Gonzalez research, UC Davis
See Gonzalez, Bombardelli, Calo and Niño, 2007

Modes of sediment transport in the Delta

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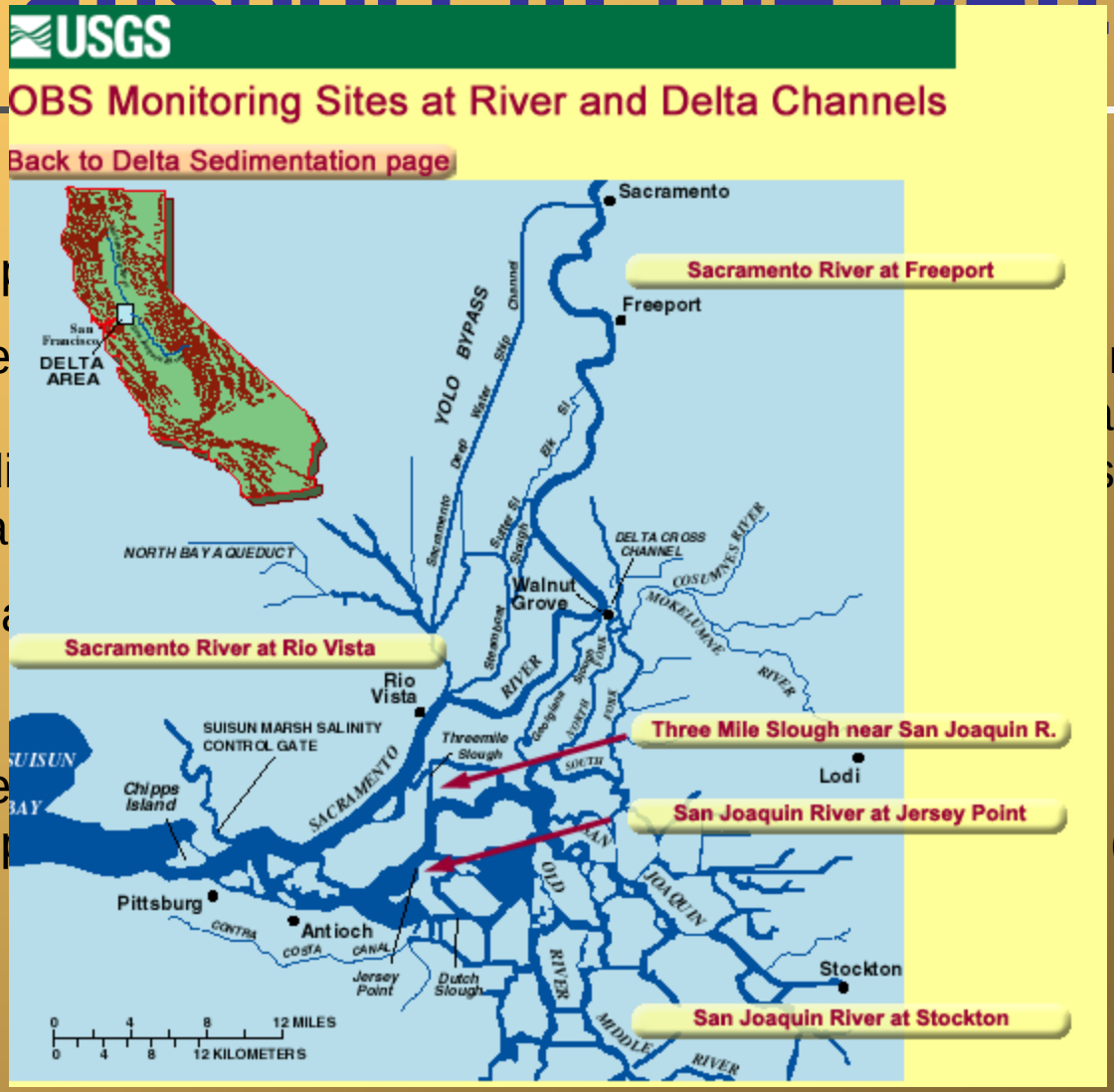
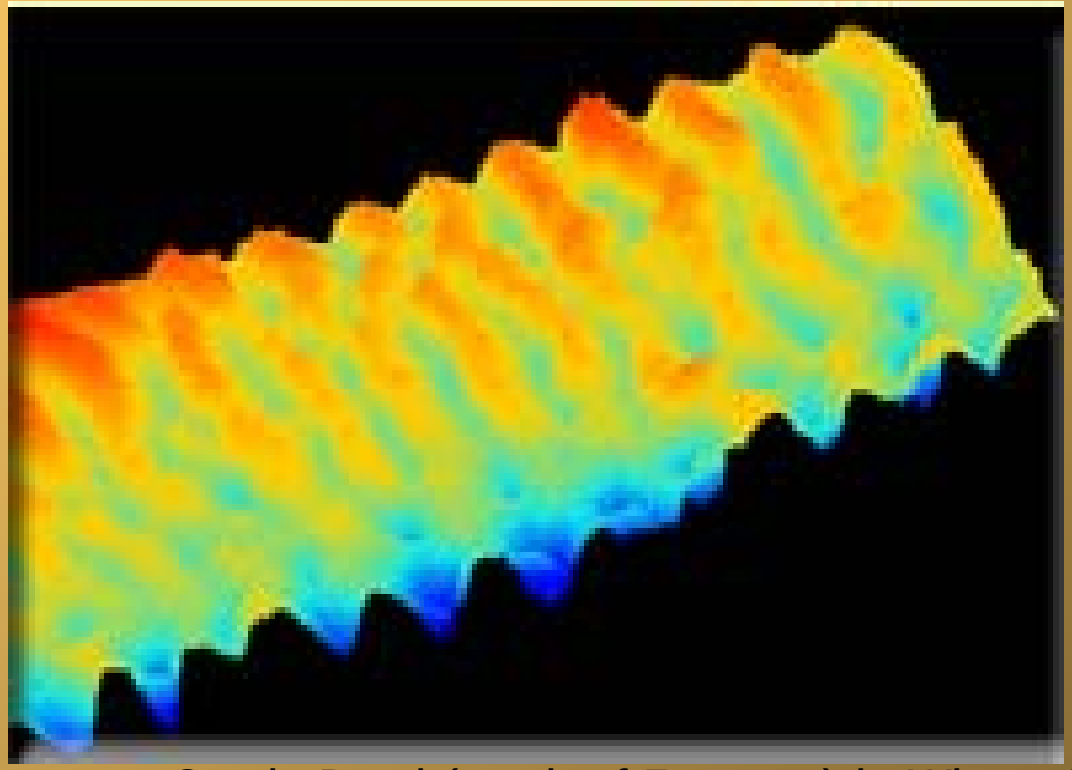


Modes of sediment transport in the Delta

We also find sand bed-forms in the Sacramento River.



Bed-forms at Garcia Bend (north of Freeport) in Winter 2000

Source: USGS website

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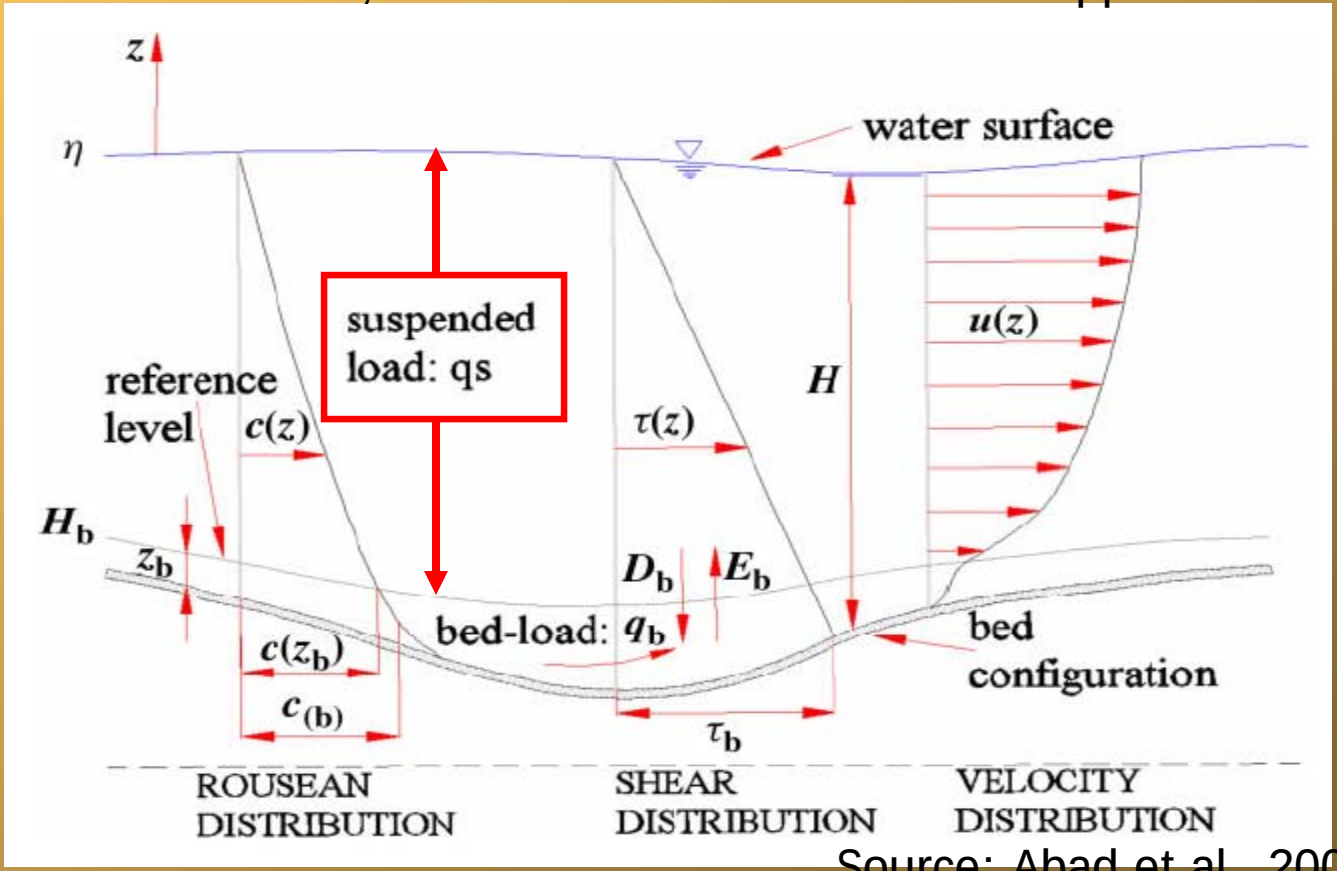
Mathematical treatment of the problem

Tracking individual particles is not feasible for a system of the size of the Delta. Then, we need to use the continuum approach.



Source: Abad et al., 2007

Mathematical treatment of the problem

Sediment transport in suspension:

$$\frac{\partial(A\,C_s)}{\partial t}+\frac{\partial(Q\,C_s)}{\partial s}=\frac{\partial}{\partial s}\left[A\,K_s\,\frac{\partial C_s}{\partial s}\right]+E-D+q_L\,C_L+S/S$$

A : cross-sectional wetted area (m^2)

C_s : volumetric cross-sectional-averaged concentration of sediment in suspension (-)

Q : flow discharge (m^3/s)

S/S : non-point sources/sinks (m^2/s)

K_s : dispersion coefficient (m^2/s)

E *and* D : entrainment rate of sediment into suspension and deposition rate of sediment per unit width, respectively (m^2/s)

q_L *and* C_L : lateral discharge (m^2/s), and concentration (-), respectively

Mathematical treatment of the problem

Predictors for sediment transport as bed-load:

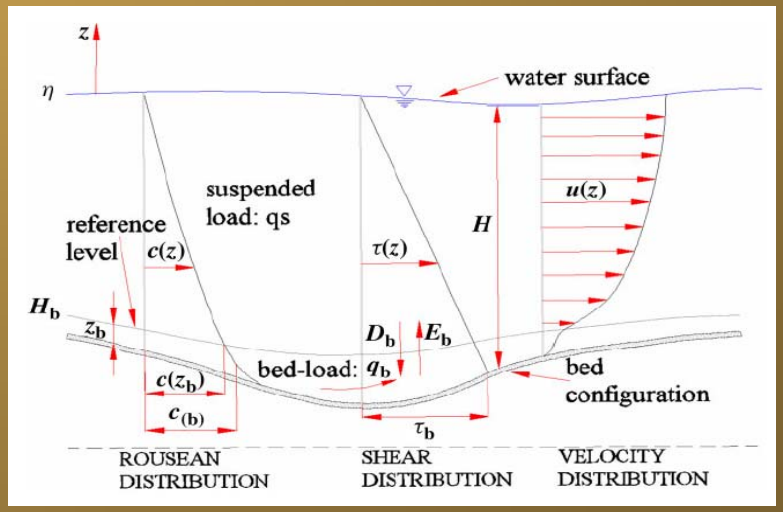
$$\frac{q_b}{\sqrt{R\ g\ d_P^3}} = f\left(excess\ shear\ stress\right)$$

q_b : bedload solid discharge per unit width (m^2/s)

$$R = \frac{\left(\rho_s - \rho\right)}{\rho}$$

d_p : sediment particle diameter (m)

g : acceleration of gravity (m/s^2)



Mathematical treatment of the problem

Changes in bed elevation (Exner equation):

$$B\frac{\partial \xi}{\partial t} = -\frac{1}{(1-\lambda_p)}\left[\frac{\partial Q_b}{\partial s} - E + D\right]$$

Q_b : total bed-load solid discharge (m^3/s)

ξ : location of the water-sediment interface (m)

λ_p : porosity of the bed sediments (-)

B : active width of the cross section (m)

The equation for sediment in suspension comes from the integration in the cross section up to z_b.

Mathematical treatment of the problem

Closures for entrainment of non-cohesive sediment:

$$E_s = \frac{A_{GP}\ Z_u^5}{1 + \frac{A_{GP}}{0.3}\ Z_u^5} \quad ; \quad Z_u = \frac{u_*}{w_s}\ f(\mathrm{Re}_p) \quad ; \quad \mathrm{Re}_p = \frac{\sqrt{g\ R\ d_p^{\ 3}}}{\nu}$$

u_* : shear (wall-friction) velocity (m/s)

$$E = E_s\ w_s\ B \quad ; \quad D = C_{sl}\ w_s\ B$$

C_{sl} : sediment concentration at the bottom (-) $= C_s\ r_0$

Mathematical treatment of the problem

Closures for entrainment of cohesive sediment:

$$E - D = C_{sl}\ w_s\ B\ (1 - \frac{U^2}{U_{cd}^2}) \quad if \quad U \le U_{cd}$$

$$E - D = M\ w_s\ B\ (1 - \frac{U^2}{U_{ce}^2}) \quad if \quad U \ge U_{ce}$$

M : erodibility coefficient

U_{cd} : critical velocity for deposition

U_{ce} : critical velocity for erosion

Mathematical treatment of the problem

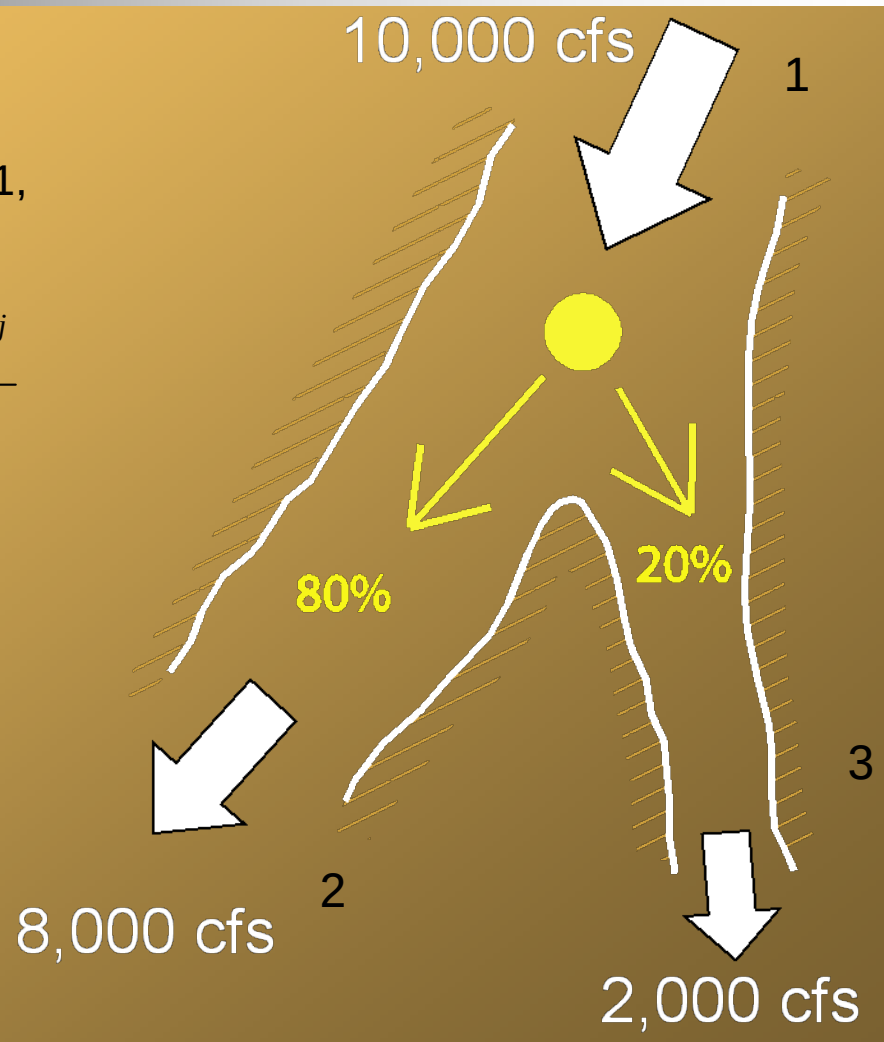
- The distribution of sediment can not be predicted in 1-D. A "closure" is thus needed (Mike 11, DHI).

$$Q_{sT,p} = \frac{\left(K_p\, Q_p^{n_p}\right) \sum_{j=1}^{NIN} K_j\, Q_{sT,j}^{n_j}}{\sum_{j=1}^{NOUT} K_j\, Q_j^{n_j}}$$

$$Q_{sT,2} = \frac{\left(K_2\, Q_2^{n_2}\right) K_1\, Q_{sT,1}^{n_1}}{K_2\, Q_2^{n_2} + K_3\, Q_3^{n_3}}$$

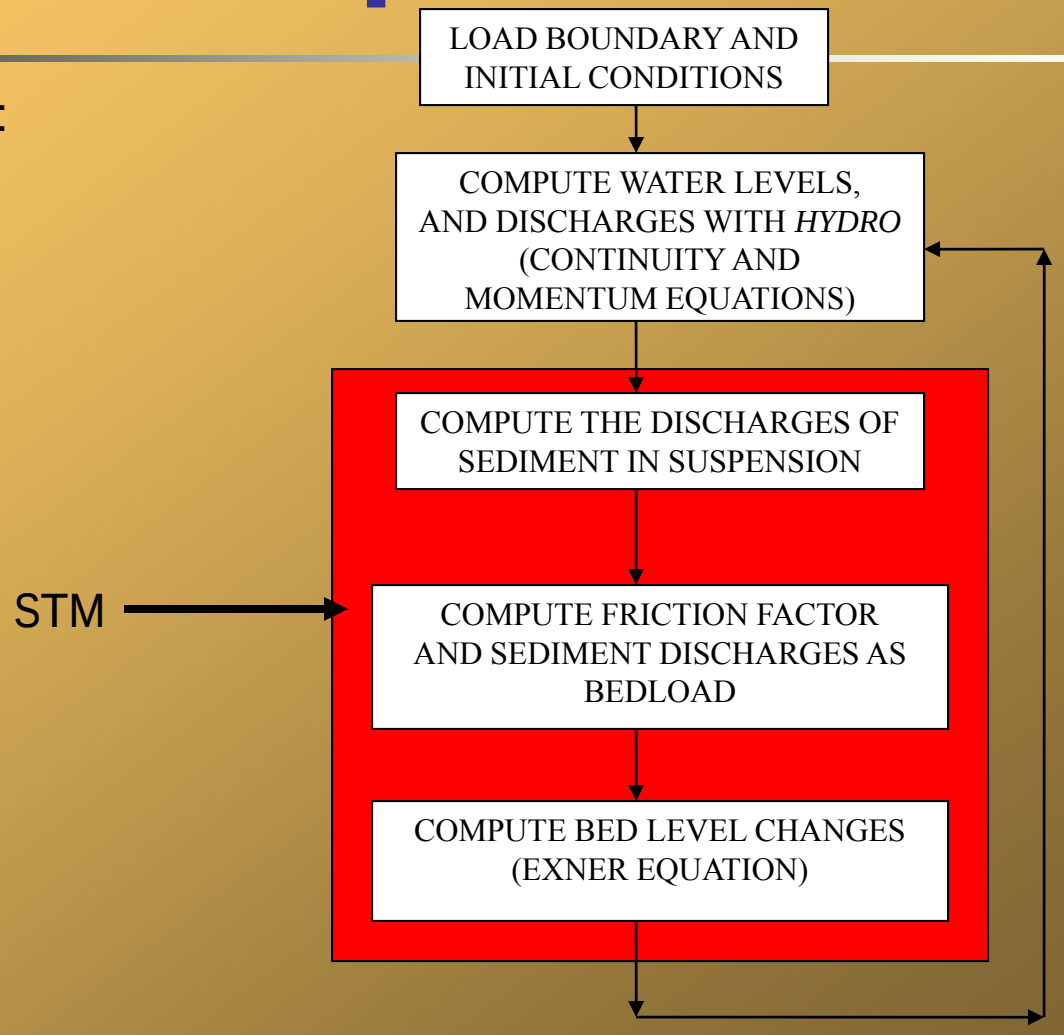
$$Q_{sT,3} = \frac{\left(K_3\, Q_3^{n_3}\right) K_1\, Q_{sT,1}^{n_1}}{K_2\, Q_2^{n_2} + K_3\, Q_3^{n_3}}$$

Similar treatment for gates.



Mathematical treatment of the problem

Flow chart:



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Numerical treatment of the problem

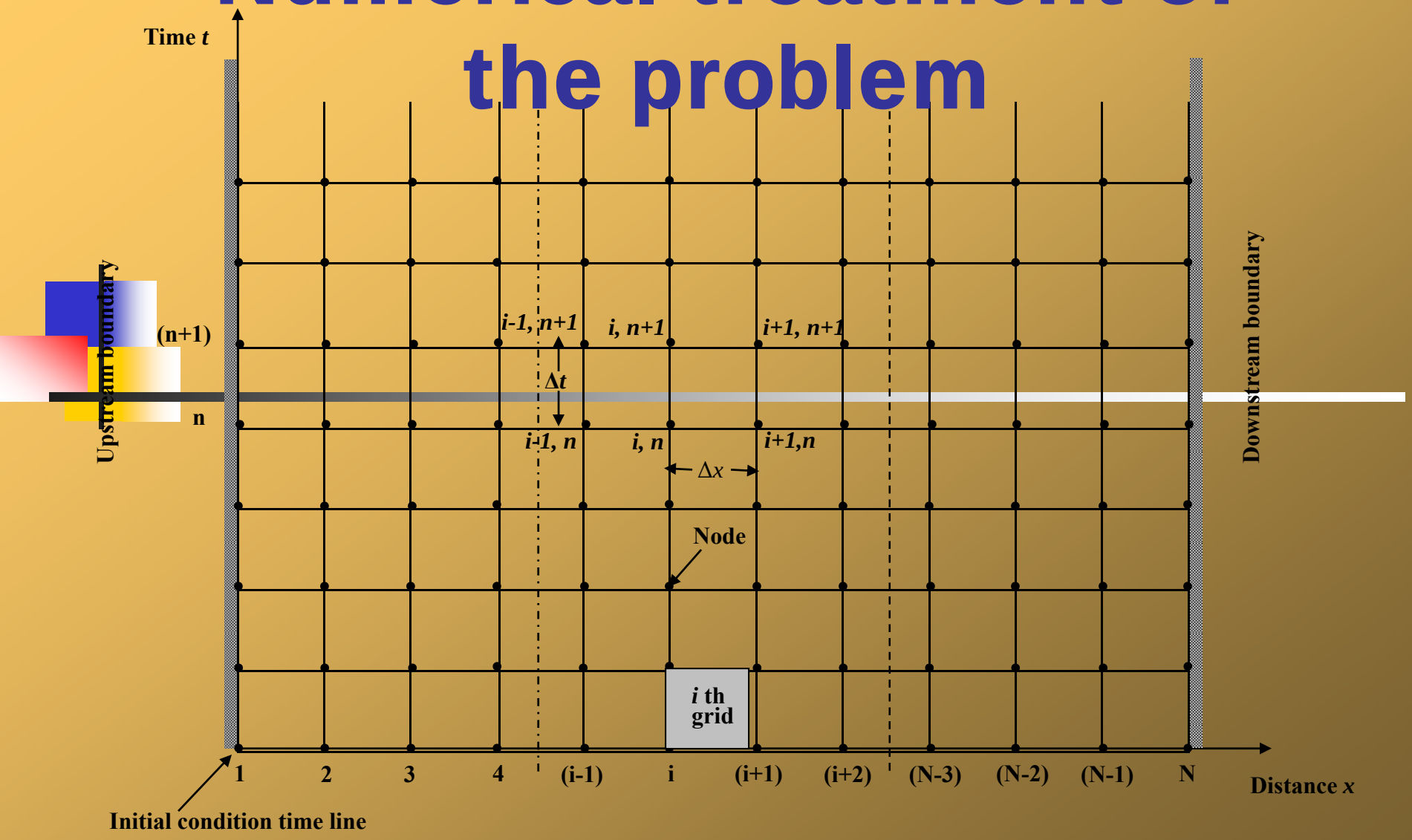
STM will feed from the results of Hydro. Then, we have to adopt a numerical scheme of at least the same accuracy of that embedded in Hydro.Hydro uses a 4-point scheme, which are different manifestations of the famous Preismann scheme. It is also implicit.

We plan to treat the transport equation in suspension with a second-order finite-volumes/finite-differences approach.

We base the numerical scheme on **splitting** the dispersion from the advection, and treat dispersion with a weighted, implicit scheme between n and n+1.

The advective term is treated with a Flux-Based, Modified Method of Characteristics (FB-MMoC), as suggested by Roache (1992).

Numerical treatment of the problem



Numerical treatment of the problem

Dispersion

$$\frac{\partial(AC_s)}{\partial t} = \frac{\partial}{\partial x}(AK_s \frac{\partial C_s}{\partial x})$$

$$\left\{ \frac{(AC_s)_i^{n+1} - (AC_s)_i^n}{\Delta t} = \theta \left[\frac{(AK_s)_{i+1/2}^{n+1} (\frac{\partial C_s}{\partial x})_{i+1/2}^{n+1} - (AK_s)_{i-1/2}^{n+1} (\frac{\partial C_s}{\partial x})_{i-1/2}^{n+1}}{\Delta x} \right] + (1-\theta) \left[\frac{(AK_s)_{i+1/2}^{n} (\frac{\partial C_s}{\partial x})_{i+1/2}^{n} - (AK_s)_{i-1/2}^{n} (\frac{\partial C_s}{\partial x})_{i-1/2}^{n}}{\Delta x} \right] \right.$$

$$\begin{cases} (\frac{\partial C_s}{\partial x})_{i+1/2}^t = \frac{(C_s)_{i+1}^t - (C_s)_i^t}{\Delta x} \\ (\frac{\partial C_s}{\partial x})_{i-1/2}^t = \frac{(C_s)_i^t - (C_s)_{i-1}^t}{\Delta x} \end{cases}$$

$$\left(\frac{-\theta\Delta t}{\Delta x^2}(AK_s)_{i-1/2}^{n+1} \quad A_i^{n+1} + \frac{\theta\Delta t}{\Delta x^2}(AK_s)_{i+1/2}^{n+1} + \frac{\theta\Delta t}{\Delta x^2}(AK_s)_{i-1/2}^{n+1} \quad \frac{-\theta\Delta t}{\Delta x^2}(AK_s)_{i+1/2}^{n+1} \right)_{1\times 3} \times \begin{pmatrix} C_{i-1}^{n+1} \\ C_i^{n+1} \\ C_{i+1}^{n+1} \end{pmatrix}_{3\times 1} =$$

$$\left((AC_s)_i^n + \frac{(1-\theta)\Delta t}{\Delta x^2} \left\{ (AK_s)_{i+1/2}^n (C_s)_{i+1}^n - (AK_s)_{i+1/2}^n (C_s)_i^n - (AK_s)_{i-1/2}^n (C_s)_i^n + (AK_s)_{i-1/2}^n (C_s)_{i-1}^n \right\} \right)_{1\times 1}$$

Numerical treatment of the problem

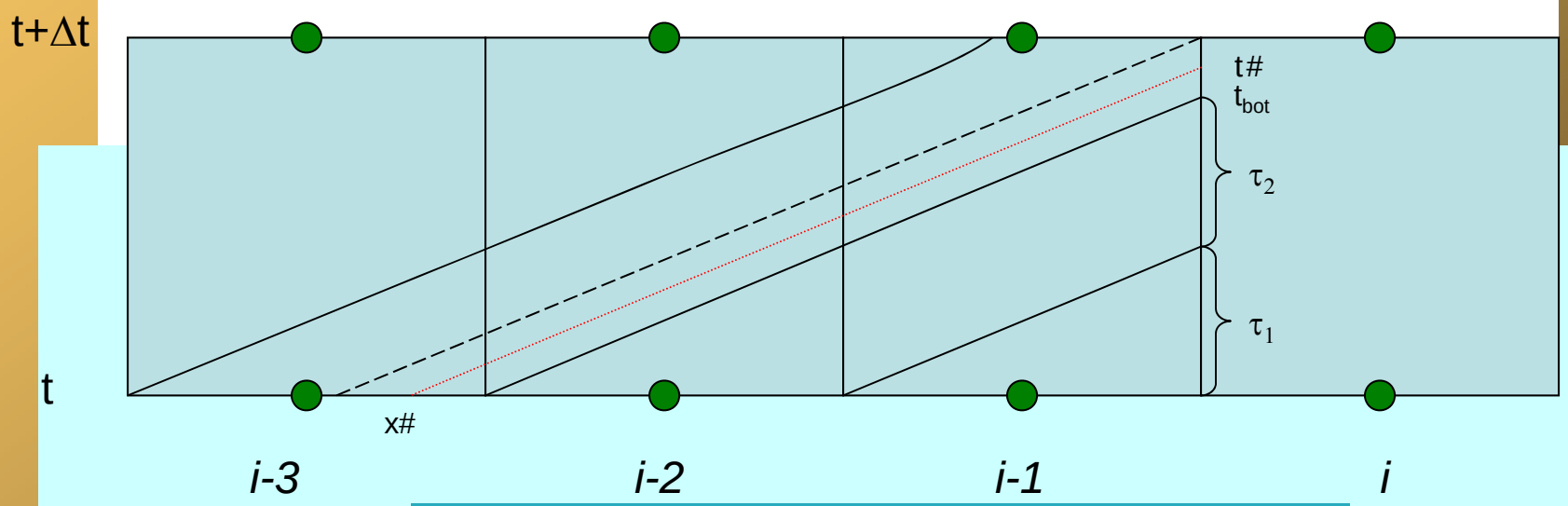
$$\frac{\partial(AC_s)}{\partial t} = -\frac{\partial}{\partial x}(U\,AC_s)$$

Advection

$$\int_t^{t+\Delta t}\int_{x-\Delta x/2}^{x+\Delta x/2}\frac{\partial(C_s\,A)}{\partial t}dx\,dt = \int_t^{t+\Delta t}\int_{x-\Delta x/2}^{x+\Delta x/2}-\frac{\partial}{\partial x}(AC_s\,U)dx\,dt$$

$$AC_s(x,t+\Delta t) = AC_s(x,t) + \frac{-1}{\Delta x}\int_t^{t+\Delta t} AC_s(x+\frac{\Delta x}{2})U(x+\frac{\Delta x}{2})dt + \frac{1}{\Delta x}\int_t^{t+\Delta t} AC_s(x-\frac{\Delta x}{2})U(x-\frac{\Delta x}{2})dt$$

$$AC_i^{n+1} = AC_i^n - \frac{\Delta t}{\Delta x}(U_R\,AC_R^{\#} - U_L\,AC_L^{\#})$$



Numerical treatment of the problem

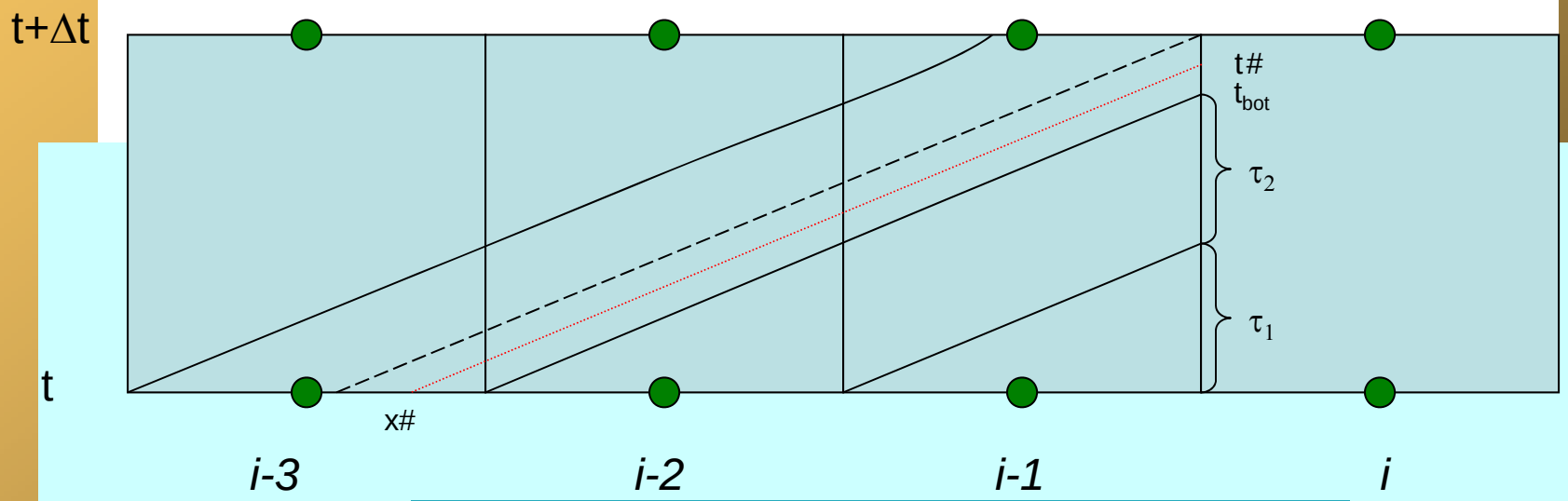
Advection

$$AC_i^{n+1} = AC_i^n - \frac{\Delta t}{\Delta x}(U_R AC_R{}^{\#} - U_L AC_L{}^{\#})$$

CFL<1

$$AC_{S\ i+1/2}^{n+1/2} = AC_{S\ i}^n + \frac{\Delta t}{2}\left(\frac{\partial AC_s}{\partial t}\right)\Big|_i^n + \frac{\Delta x}{2}\left(\frac{\partial AC_s}{\partial x}\right)\Big|_i^n \qquad \frac{\partial AC_s}{\partial t} = -\frac{\partial QC_s}{\partial x}$$

$$(AC_{s\#R})_{i+1/2}^{n+1/2} = AC_{S\ i}^n + \frac{\Delta t}{2}\left(-\frac{\partial QC_s}{\partial x}\right) + \frac{\Delta x}{2}\left(\frac{C_{s\ i+1}^n + C_{s\ i-1}^n}{2\Delta x}\right)$$



ΔX=constant, CFL=$\Delta x/\tau_1$, $X^{\#} = X_{i-1/2}-(CFL \times t\#)$

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Proposed work

I Initial stage – No bed changes in the streams of the Delta; no bed-load transport

Focus on:

1) Sediment in suspension.
2) A single particle size.
3) Deposition of sediment as a net effect.
4) Cohesive sediments.

II Second stage – Several sizes of sediment in suspension.

III Add bed-load transport and entrainment functions.

IV Add bed changes.

Proposed work and timeline. *Validation*

Validation will involve two stages:

1) Comparison with analytical solutions for the advection, dispersion and advection-dispersion.
2) Comparison with laboratory experiments involving sediment transport:
- Newton (1951);
- Soni et al. (1980, 1981);
- Cui et al. (1996).

- Newton (1951): flume 9.14 m long, 1 ft wide and 2 ft deep. Sand of 0.69 mm of mean size was used.
- Cui et al. (1996): flume 45 m long, 1 ft wide.

Proposed work and timeline. *Validation*

Validation with data pertaining to the Delta:

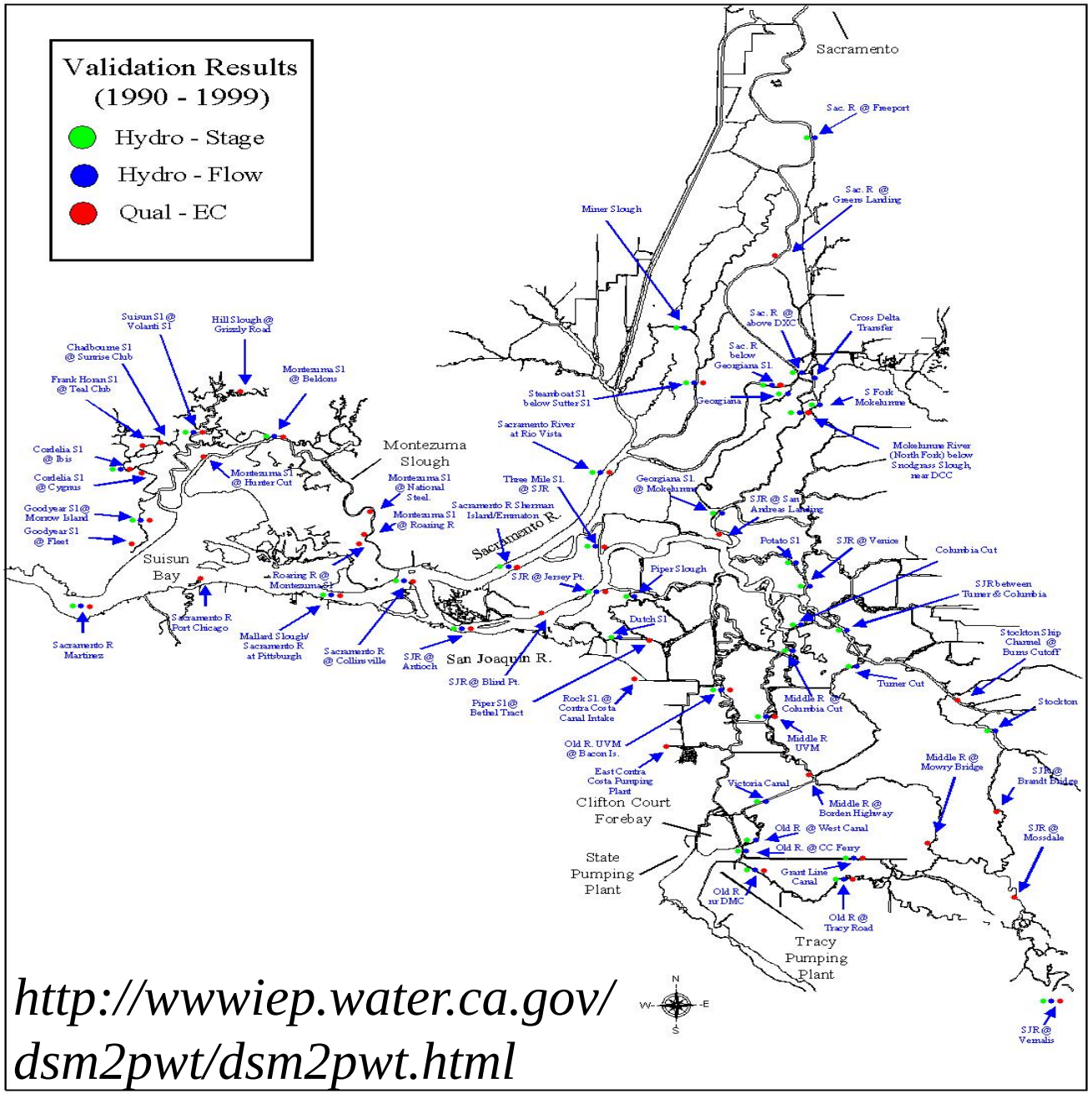
These scenarios will be determined from the inventory, possibly years 1998 to 2000.

Scenarios to simulate when the code is finished:

1) Dredging activities in the Stockton ship canal.
2) Marsh restoration in the rivers of the network.
3) Levee breaches at convenient locations.

Calibration and validation info available on the Web

Courtesy: Tara Smith and Jamie Anderson



http://wwwiep.water.ca.gov/dsm2pwt/dsm2pwt.html

Future work

Link of Hydro with STM and QUAL, to simulate the transport and fate of metals for instance.

- Equation for the dissolved phase in the river:

$$\frac{\partial(C_i)}{\partial t}+\frac{\partial(UC_i)}{\partial x}=\frac{1}{A}\frac{\partial}{\partial x}\left(D_h A\frac{\partial C_i}{\partial t}\right)+k_b\frac{P}{A}\left(c_i(0)-C_i\right)-C_s k_r\left(K_D C_i-S_i\right)+R_i+$$

$$\frac{q_{in}}{A}(C_{i,in}-C_i)+k_{at}\left(C_{i,at}-C_i\right)$$

- Equation for the particulate phase in the river:

$$\frac{\partial C_s S_i}{\partial t}+\frac{\partial UC_s S_i}{\partial x}=\frac{1}{A}\frac{\partial}{\partial x}\left(D_s A\frac{\partial C_s S_i}{\partial x}\right)+Er.w_p.\frac{P}{A}s_i(0)-w_p r_0.\frac{P}{A}.C_s S_i+$$

$$\frac{q_{in}}{A}\left(C_{s,in}S_{in}-C_s S_i\right)+k_r C_s\left(K_D C_i-S_i\right)$$

Source: Massoudieh, Bombardelli and Ginn, 2009

Conclusions

- Sediment-transport subroutines will be incorporated to DSM2, through a new module called STM.
- The new module will solve equations for the sediment in suspension, for the transport of sediment as bed-load, and for the change in bed elevation, in diverse sub-modules.
- Numerical schemes for the new module will be based on a finite-volumes/finite-differences scheme. The dispersive term will be treated in an implicit way which is second-order in both time and space for a Crank-Nicolson situation. The advective term will be treated with a Flux-Based, Modified Method of Characteristics (FB-MMoC), which is also second order in space. Cases of Courant numbers larger and smaller than one will be included. Flux limiters will diminish the presence of wiggles.
- The project will also provide an inventory of datasets of sediment transport in the Delta.